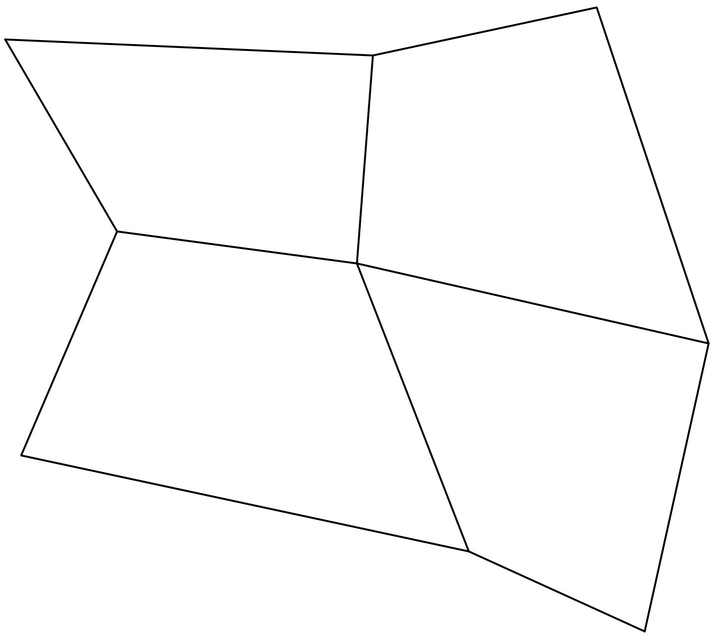
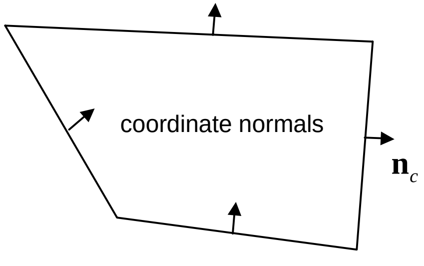
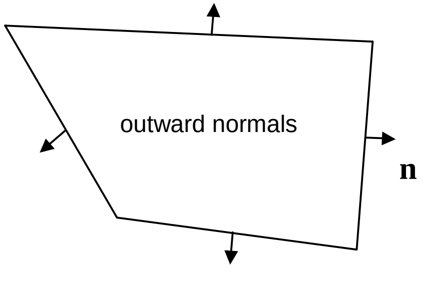
Chevron Flow Scenario

Consider a logically rectangular, but non-orthogonal mesh **M**:

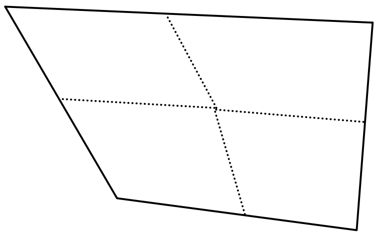


The centers of cells, faces, and edges are at the centroids of these elements. Also define two sets of normals (at the face centers):





Define a refined mesh **S** built from the cell centers, face centers and vertices of the original mesh:



Define the following fields:

- The pressure P, a scalar field, cell-centered on mesh **M**.
- The permeability $\overleftrightarrow{\mathbf{K}}$, a tensor field, cell-centered on mesh **M**.
- The pressure gradient ∇P, a vector field, cell-centered on mesh **S**.
- The flux q, a scalar field, multi-face-centered on mesh **M**.

The flux is defined:

$$q = \overleftrightarrow{\mathbf{K}} \cdot \nabla P \cdot \mathbf{n}_c$$

where all quantities are evaluated at the multi-face centers.

Multi-face centering means that the centering points are located at the faces of **S** that coincide with the faces of **M**.

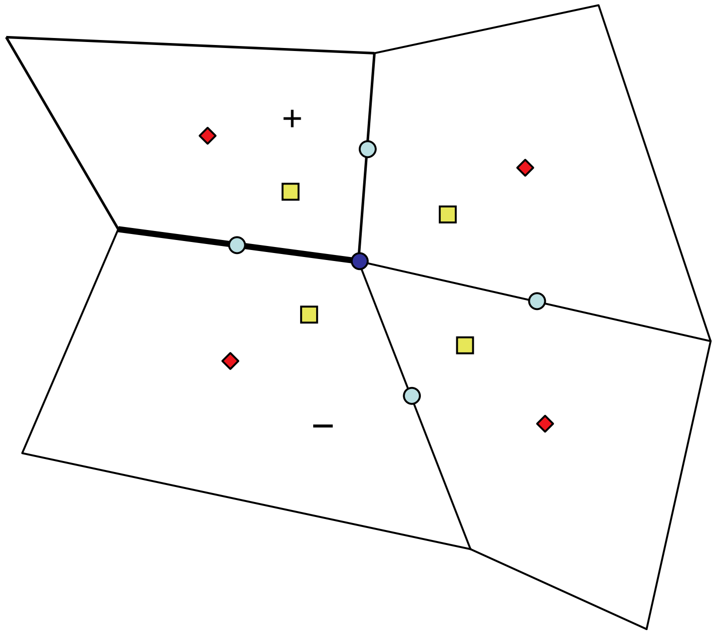
Let us assume that we know the pressure and permeability everywhere.

We would like to compute the total flux of material leaving every cell. A procedure taken from a paper by Lee, Tchelepi, and Dechant is as follows:

For each vertex **V** in **M**:
    For each face **F** in **M** that contains **V**
        Write down a pressure continuity equation at the face center
    For each face **G** in **S** that contains **V**
        Write down a flux continuity equation at the face center
    Gather the pressure and flux continuity equations and solve for the pressure gradient values in the cells in **S** that contain **V**.
    For each face **H** in **S** that contains **V**
        Compute the flx

For each cell **C** in **M**:
    Set the total flux out the cell Q to 0
    For each face **H** in **M** that bounds **C**
        For each face **I** in **S** contained in **H**
            Accumulate fluxes into a temporary Q_H
        Compute a sign s indicating whether or not the coordinate normal aligns with the outward normal for **H**
        Multiply Q_H by s and the area of **H** and accumulate into Q

Pressure Continuity Equation:



Vertex **V**

Face center **f**

Pressure gradient value (and cell center $\mathbf{c_S}$)

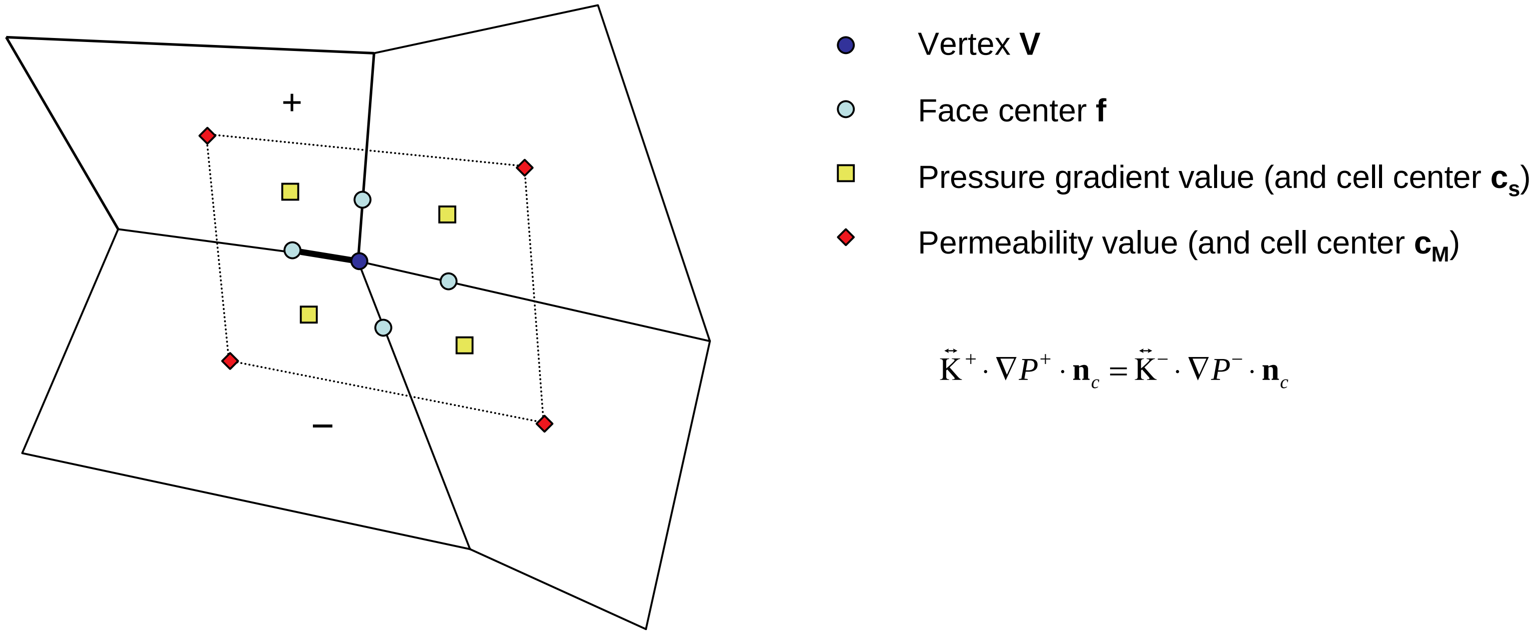
Pressure value (and cell center $\mathbf{c_M}$)

$$P^{+} + \nabla P^{+} \cdot \left(\mathbf{x}_f - \mathbf{x}^{+}_{c_M}\right) = P^{-} + \nabla P^{-} \cdot \left(\mathbf{x}_f - \mathbf{x}^{-}_{c_M}\right)$$

The + and – superscripts refer where a value is relative to the face. If it lies in the direction of the coordinate normal, the orientation is “positive.”

The cell-centered values selected are those nearest the reference vertex.

Flux Continuity Equation:

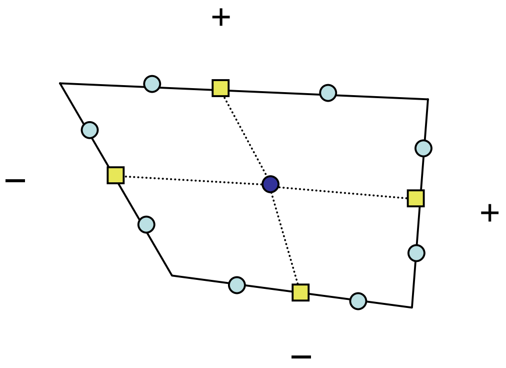


$$\overleftrightarrow{K}^{+} \cdot \nabla P^{+} \cdot \mathbf{n}_c = \overleftrightarrow{K}^{-} \cdot \nabla P^{-} \cdot \mathbf{n}_c$$

The + and – superscripts refer where a value is relative to the face. If it lies in the direction of the coordinate normal, the orientation is “positive.”

The cell-centered values selected are those nearest the reference vertex.

Flux Accumulation:



Cell **C**

Sub-face center **sf**

Face center **f**

$$Q = \sum_{faces} s_f A_f \sum_{sub-faces} q_{sf}$$

The + and – superscripts refer whether or not the sign *s* is positive or